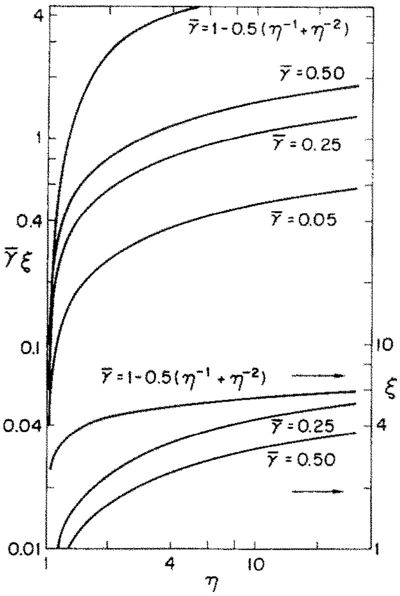

FIG. 2. Dimensionless heat sink profiles $\bar{\gamma}\xi$ and dimensionless film profiles ξ.

For the constant heat flux case $\bar{\gamma}$, would be less than this value. The limit for the value of $(T_{lv}-T_v)_0$ depends on the ability of the evaporating film profile to change and provide the necessary pressure gradient for flow which is finite.

In Fig. 2 the dimensionless heat sink capability $\bar{\gamma}\xi$ is presented as a function of the dimensionless thickness η. The value of the heat sink for the constant liquid–vapor interfacial temperature case starts below the constant heat flux case because of effect of the dispersion force on vapor pressure. However, it surpasses the constant heat flux case at a very small dimensionless thickness. The thin film profiles for three values of $\bar{\gamma}$ are also presented in Fig. 2. Combining the results of the two limiting cases for a closer approximation of the real physical situation, we can say, qualitatively, that the initial portion of the evaporating thin film follows the constant temperature model until thermal resistance limits the heat flux to a constant value. This limit occurs at a very small increase in film thickness.

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NOTE ON DIFFUSION IN A TURBULENT BOUNDARY LAYER

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NOMENCLATURE

C_f, skin friction coefficient;
c_p, specific heat at constant pressure;
k, effective turbulent Prandtl number near wall;
l, characteristic length of source;
q, strength of heat source per unit length;
u, velocity in x direction;
x, distance from the origin of the turbulent boundary layer;
x', distance from the source;
y, distance normal to the wall;
$y_{1/2}$, distance where $\theta = \theta_w/2$;
$\bar{y}$, mean diffusion distance normal to the wall.

Greek symbols
δ^*, boundary layer displacement thickness;
ε, eddy diffusivity;
θ, temperature difference referenced to the free-stream temperature;
ν, kinematic viscosity;
ρ, density;
τ_w, wall shear stress.

Subscripts
0, at the source;
w, at the wall;
∞, in the free stream.

THIS note concerns one aspect of turbulent flow which has received repeated attention—diffusion in a turbulent boundary layer downstream of a line source of heat (or mass) positioned on an adiabatic (or impervious) surface. In particular, two existing analyses of this problem which use substantially different eddy diffusivity distributions are compared to published experimental results. The purpose of the note is to show that these theories successfully describe the mean temperature (or concentration) distribution downstream of the source providing a near and far field approach is taken.

Consider a line source of heat placed on an adiabatic wall and at right angles to the flow. If the characteristic source dimension l, is less than the linear sublayer thickness and the buoyancy force near the source is small compared to the inertia force in the sublayer, i.e. the source Grashof number is much less than $l^2\tau_w/\rho\nu^2$, where $(\tau_w/\rho)^{1/2}$ and ν are the friction velocity and kinematic viscosity, respectively, the flow downstream of the source can be considered to develop as an undisturbed turbulent boundary layer.

Near the source, diffusion is controlled by the turbulence structure near the wall and providing the source is located

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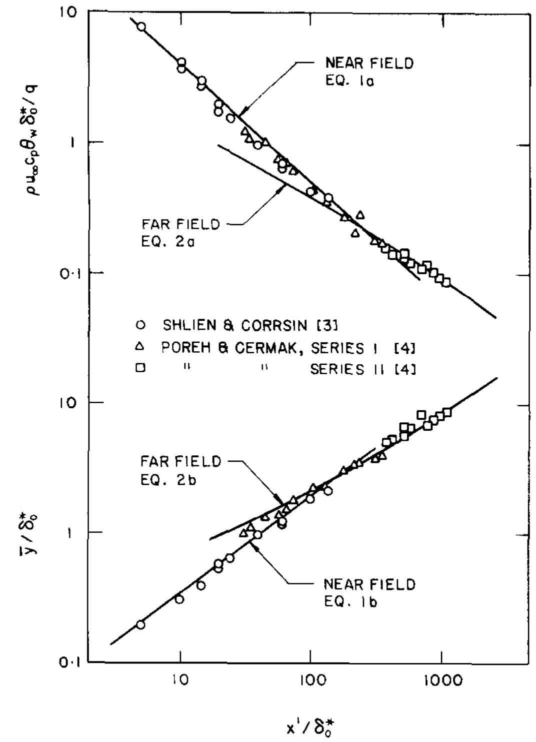

FIG. 1. Streamwise variation of the wall temperature (or concentration) and diffusion distance.

some distance downstream from the origin of the shear layer the convection of heat normal to the wall can be neglected compared to that along the surface. A solution to this problem was found by Frost [1]. He considered diffusion from an infinite line source within the constant shear stress region of a turbulent boundary layer having a uniform thickness. Although Frost assumed a particular mixing length distribution, his result can also be obtained by assuming an eddy diffusivity proportional to an eddy viscosity and solving for the latter from his stress distribution by using a power law velocity profile. Since Frost considered the shear stress constant, his analysis is limited to diffusion within the wall region of the boundary layer and can be expected to predict a diffusion rate which is too large far from the source. Hence, this analysis may be said to apply only in the *near field*, i.e. near the source.

In contrast, far from the source, diffusion within the wake region of the boundary layer is important and the convection of heat normal to the wall cannot be neglected. An analysis of this problem was recently presented by Mayle and Kopper [2]. They assumed that the eddy diffusivity is a function of the streamwise distance alone. This was based on the observation that both near the wall and at the edge of the thermal layer where the diffusivity can be expected to change most rapidly, the temperature gradients are nearly zero by virtue of the adiabatic wall and freestream constraints. Therefore, since any error in the eddy diffusivity within these regions is of little consequence, the diffusivity may be taken as independent of the distance from the wall. Obviously, for a heat source imbedded in the wall, this assumption is not appropriate until the thermal layer has at least diffused into the wake region of the shear layer. Hence, this theory may be said to apply only in the *far field*, i.e. far from the source, and can be expected to predict a diffusion rate which is too large near the source.

In a recent article (which stimulated the present comparison) Shlien and Corrsin [3] presented dispersion measurements in a zero pressure gradient turbulent boundary layer downstream of a long heated wire placed parallel to a wall and normal to the flow. These complement some earlier turbulent boundary layer measurements of Poreh and Cermak [4] on diffusion from a line source of mass positioned at two streamwise locations (series I and series II tests) on an impervious wall. As will be seen, the data from these references provide information within both the near and far field.

The comparisons between theory and measurements will be made by first presenting the streamwise variation of the adiabatic wall temperature θ_w, and mean diffusion distance, $\bar{y}$, normal to the wall. For a heat source of strength q placed where the boundary layer displacement thickness is δ_0^*, a dimensionless adiabatic wall temperature and mean diffusion distance may be defined as

$$\frac{\rho u_\infty c_p \theta_w \delta_0^*}{q} = \frac{u_\infty \theta_w \delta_0^*}{\int_0^\infty u\theta \, dy} \quad \text{and} \quad \frac{\bar{y}}{\delta_0^*} = \frac{\int_0^\infty y\theta \, dy}{\delta_0^* \int_0^\infty \theta \, dy}$$

respectively,‡ where u is the velocity and θ is the temperature difference referenced to the free-stream temperature at the distance y from the wall, ρ is the density and c_p the specific heat at constant pressure. From the solutions given in [1] and [2] for a one-sixth power velocity profile, which fits the measured profiles in [3] and [4], it is not difficult to obtain the following theoretical expressions for the streamwise variation of these quantities:

Near field

$$\frac{\rho u_\infty c_p \theta_w \delta_0^*}{q} = 0.121\left(\frac{1}{k}\frac{C_{f0}}{2}\frac{x'}{\delta_0^*}\right)^{-7/8} \tag{1a}$$

$$\frac{\bar{y}}{\delta_0^*} = 6.94\left(\frac{1}{k}\frac{C_{f0}}{2}\frac{x'}{\delta_0^*}\right)^{3/4} \tag{1b}$$

where

$$k = \frac{1}{\varepsilon}\frac{\tau_w}{\partial u/\partial y}$$

is considered a constant, ε the eddy diffusivity, and x' is the streamwise distance from the source. For turbulent flow along a flat plate the skin friction coefficient, C_{f0}, can be obtained from

$$\frac{C_{f0}}{2} = 0.0140\left(\frac{u_\infty \delta_0^*}{\nu}\right)^{-1/4}$$

Far field

$$\frac{\rho u_\infty c_p \theta_w \delta_0^*}{q} = 0.253\left(1+\frac{x'}{x_0}\right)^{-4/5} \times \left[1-\left(1+\frac{x'}{x_0}\right)^{-52/35}\right]^{-7/13} \tag{2a}$$

$$\frac{\bar{y}}{\delta_0^*} = 2.97\left(1+\frac{x'}{x_0}\right)^{4/5}\left[1-\left(1+\frac{x'}{x_0}\right)^{-52/35}\right]^{6/13} \tag{2b}$$

where the turbulent Prandtl number has been taken as 0.65 and x_0 is the distance to the source from the origin of the turbulent boundary layer, viz.

$$\frac{x_0}{\delta_0^*} = (0.0494)^{-5/4}\left(\frac{u_\infty \delta_0^*}{\nu}\right)^{1/4}.$$

The first comparisons are shown in Fig. 1. Since the test wall in [3] was not truly adiabatic, only the data where the heat loss was minimal is shown. The Reynolds number based on δ_0^* was 6300 for the test in [3] and the series I test in [4]. This

‡ The analogy between mass and heat transfer is implied throughout even though the notation is specifically for a heat source. A turbulent Lewis number of unity is implied in Figs. 1 and 2.

value was used for the near field calculations. For the far field calculations, the average value for the series I and II tests in [4] of 4600 was used. As shown in Fig. 1, the near field theory, equations (1a) and (1b), correlate the data for distances, x'/δ_0^*, up to about 200. It was found that $k = 0.87$ provided the best fit to the θ_w and $\bar{y}$ data combined. If k is interpreted as an average turbulent Prandtl number within the wall region, its value is quite acceptable. However, it remains to determine whether or not k depends on the Reynolds number based on δ_0^*. For $x'/\delta_0^* > 200$, the actual diffusion rate is less than that calculated by the near field equations, but is correlated reasonably well by the far field theory, i.e. equations (2a) and (2b). In this region, it is interesting that the turbulent Prandtl number of 0.65 which was determined in [2] from an independent experiment appears satisfactory for these experiments also.

A second, although somewhat qualitative comparison is presented in Fig. 2. The bands enclose the measured temperature profiles reported in [3] and [4] (series II) and represent the near and far field data, respectively. The theoretical mean temperature profiles, which are similar within their respective regions but different than one another, are

$$\frac{\theta}{\theta_w} = \exp[-0.693(y/y_{1/2})^{4/3}] \tag{3}$$

in the near field [1] and

$$\frac{\theta}{\theta_w} = \exp[-0.693(y/y_{1/2})^{13/6}] \tag{4}$$

in the far field [2] where $y^{1/2}$ is the value of y at $\theta = \theta_w/2$. These profiles are plotted in Fig. 2 and fall within their corresponding data bands except that for the near field close to the wall.

In [5], Morkovin concluded that observed characteristics of a layer diffusing within a pre-established turbulent shear layer could be accounted for by considering the eddy diffusivity a property of a quasi-similar field. Apparently, provided it is only necessary to describe the mean temperature (concentration) distribution downstream of a heat (mass) source on an adiabatic (impervious) surface, a rather simple two part field is sufficient.

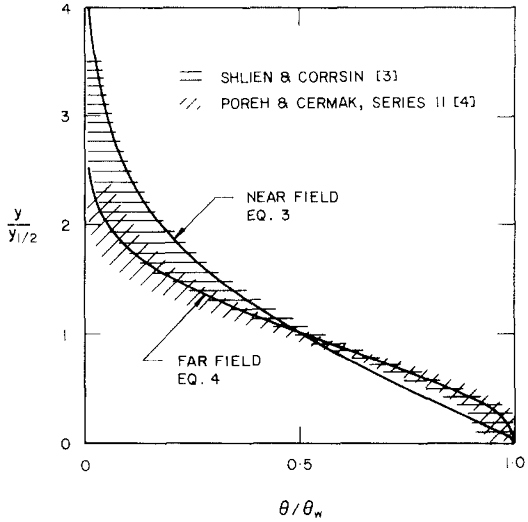

FIG. 2. Temperature (or concentration) profiles.

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